

BULLETIN
of the
American Association of
Jesuit Scientists

Eastern Section
Founded 1922



Published at
CHEVERUS HIGH SCHOOL
Portland, Maine

CONTENTS

Science and Philosophy:

- Our Holy Father Speaks on Science.
 Contributed by Rev. Joseph P. Kelly, S.J., Weston College, Weston,
 Mass.192

Astronomy:

- Some Problems in Statistical Astronomy.
 Rev. Francis J. Heyden, S.J., St. Mary's Rectory, Boston, Mass. 198

Biology:

- A Few Vitamin Complexes:
 James J. Deeley, S.J., Weston College, Weston Mass. 207

How Do You Explain It? 212

Mathematics:

- Nth Deratives of finite Products.
 C. F. Kohler, S.J., Weston College, Weston, Mass. 213

Bulletin of American Association of Jesuit Scientists

EASTERN STATES DIVISION

VOL. XXI

MARCH, 1944

No. 3

EDITORS OF THE BULLETIN

Editor in Chief, REV. GERALD F. HUTCHINSON, S.J.
Cheverus High School, Portland, Maine

ASSOCIATE EDITORS

Biology, REV. PHILIP O'NEILL, S.J.

Chemistry, REV. BERNARD FIEKERS, S.J.

Mathematics, REV. GEORGE O'DONNELL, S.J.

Physics, REV. PETER McKONE, S.J.

Science and Philosophy, REV. JOSEPH P. KELLY, S.J.

CORRESPONDENTS

Chicago Province: REV. VICTOR C. STECHSCHULTE, S.J.
Xavier University, Cincinnati, Ohio.

Missouri Province: REV. PAUL L. CARROLL, S.J.
St. Louis University, St. Louis, Missouri.

New Orleans Province: REV. GEORGE A. FRANCIS, S.J.
Loyola University, New Orleans, Louisiana.

California Province: REV. CARROLL M. O'SULLIVAN, S.J.
University of San Francisco

Oregon Province: REV. LEO J. YEATS, S.J.
Gonzaga University, Spokane, Washington

Canadian Provinces: REV. ERIC O'CONNOR, S.J.
160 Wellesley Crescent, Toronto, Canada

SCIENCE and PHILOSOPHY

ALLOCUTION OF THE HOLY FATHER, PIUS XII, TO THE
PONTIFICAL ACADEMY OF SCIENCE.* Feb. 21, 1943.

REV. JOSEPH P. KELLY, S.J.

THE IMPORTANCE OF THE QUESTION OF PHYSICAL LAWS.

In investigating the laws that govern the universe, you are in fact, moving in search of God, following the traces which He left when He completed the work of creation. And we watch with admiration the conquests which you are making in the vast fields of nature. The empirical researches of the past few decades, following closely upon the study and work of the past few centuries, can boast of discoveries and inventions of first importance,—to mention only the artificial transformation of the atomic nucleus, the splitting of the atom and the marvels of the small-scale investigation revealed by the electronic microscope. Scientific progress has led to the knowledge of new laws in natural phenomena and thrown new light on the essence and value of physical laws. There is perhaps no problem which so much absorbs the attention of the most eminent among physicists, chemists, astronomers, biologists and physiologists—and we may also say of modern students of natural philosophy—as that of the laws which regulate the order and action of the material substances and phenomena operating on our planet and in the universe. These are the fundamental questions and their solution is no less important for the scope of each science than for the science of metaphysics, which has its roots in objective reality.

CHANGES IN THE CONCEPT OF PHYSICAL LAWS, DYNAMIC AND

STATISTICAL LAWS.

A truly rigorous and dynamic law is a strict norm, so regulating the being and action of things as to exclude any exception in the natural order. Discovered by way of induction from observation and from the examination of many particular similar cases, it makes it possible to foresee, and by way of deduction, often even to calculate beforehand other particular cases, which fall within the sphere of its application; this is true, for example, of the law of gravitation, the laws of reflection and refraction of light, the law of constant proportion of atomic weights in chemical combinations and many others. But the concept of physical law has not always remained the same; and

*Note. This allocution of the Holy Father was printed in the *Acta Apost. Sedis*, XXXV, 1943, p. 70. As far as we know, this issue is not available, due to war conditions. The translation is taken from *The Clergy Review*, Nov. 1943, p. 519 sq.

it will be useful to follow the changes which have come about in its formation and valuation, in the course of the past hundred years. At the beginning of the last century, the law of conservation of mass was already known; then came the discovery of important laws of optics, electricity, and especially of physical chemistry; discoveries finally crowned by that of the general laws of energy. No wonder, then, that at the birth of materialistic monism the law of the machine was exalted as a goddess on the altar of science and that to its complete dominion, not only the whole world of matter but also the kingdom of life and spirit paid homage and allegiance. The universe now was nothing else but a boundless empire of motion, and according to this conception, as explained by DuBois-Raymond, (1), there must exist a universal mechanical formula, the knowledge of which would enable a universal genius, or "Laplace" intelligence to understand fully everything that is happening at present; nor would anything ever take him unawares, since the buried past as well as the remote future would be completely open to his gaze. The same idea was expressed by the great French mathematician, H. Poincaré: "Every phenomenon however minute, has a cause and an infinitely powerful mind, infinitely well informed of the laws of nature could have foreseen it from the beginning of ages." (2).

The postulate of a "closed, physical causality" thus admitted no exception or intervention in the course of physical activities, for example, by a miracle. But this postulate is equivalent to the old saying that, given a cause, so long as it is sufficient, the effect necessarily follows; a proposition which St. Thomas, with Aristotle showed to be false; because not every cause, even when sufficient, is such that its effect may not be prevented, at least by a free, human act. In other words, every effect has necessarily a cause, but not always a cause which operates necessarily, since there are also causes which operate freely. (3)

And yet a man of the calibre of Virchow could make the following weighty pronouncement before the 47th annual conference of German scientists and doctors in 1874: "It is certainly not an assumption on the part of natural science to assert that natural laws are absolutely effective in all circumstances, and are not subject to suspension at any time." But Virchow had not seen all the circumstances of past events or of future events either, and he was in fact making an assumption, as the scientific development of the past decades has shown. The crass materialism of those days has long been found untenable, or else has transformed itself into that darksome angel of light, (4), which appears in the guise of idealistic pantheism; while

(1). "Ueber die Grenzen des Naturerkennens." Leipzig. 1907.

(2). Poincaré, "Science and Méthode." p. 65.

(3). Cf. "In Lib. Peri Hermeneias. Lib. I, C. XI; Lect. xiv, n. 11.

(4). Eph. vi, 12. III Cor. xi, 14.

the assertion of natural laws subject to no exception whatever has been so much shaken by the progress of exact science, that people are now almost falling into the opposite excess of talking only of laws of average, statistical norms and laws of probability. This mode of thought is legitimate to the extent that many laws of the sense-world and macrocosm manifest a statistical character—because they do not express the behavior of every single being but the average process of an immense number of similar beings—and therefore show themselves amenable to the calculus of probabilities. But to see in the world nothing but statistical laws is an error of our age, while to assert that the old rigorously dynamic conception of natural law can be completely abandoned and has become meaningless, is to forswear the nature of the human mind which,

. . . through the senses doth apprehend

What then is meet for the intellect to view. (5)

Indeed a recent positivist and conventionalist trend of thought goes so far as to cast doubt upon the value of casual laws.

(N.B. On the subject of statistical laws the Holy Father also observes elsewhere in his address: "In the closely concatenated and organized system to which science has reduced the macrocosm there are doubtless many statistical laws; but these, when we consider the number of elements involved (atoms, electrons, photons, etc.), do not fall notably short of strictly dynamic laws in certainty and exactness. In any case, they are founded and, as it were, anchored in rigorously dynamic laws of the microcosm, although our particular knowledge of microcosmic laws is as yet almost non-existent, in spite of the strenuous efforts made in recent investigations with a view to understanding the mysterious activity of the inner recesses of the atom. Gradually it will be possible to lift these veils; the apparently non-causal character of microcosmic phenomena will then disappear: a new and marvelous kingdom of order, a rule of order, even in the minutest particles, will be discovered.)

WHAT IS SCIENCE?

This positivist theory is rightly rejected by sound philosophy. For what is science but the certain knowledge of things? And how is it possible to acquire this knowledge without scrutinizing the principles and causes from which the demonstration of their being, nature and actions proceeds? You observe and study nature, you carry out experiments and researches, in order to understand the principles and intrinsic grounds of nature, to reach the laws which regulate its constitution and behavior, to formulate the process of these laws and to deduce from them a science with principles, causes and conclusions which follow by logical consequence. You are therefore seeking for regularity and order in the various kingdoms of creation; and what riches the mind of man has discovered therein.

(5). Par. IV. 41-2.

THE OBJECTIVE REALITY OF KNOWLEDGE.

This marvelous, intricate and orderly system of laws, qualitative and quantitative, particular and general, macrocosmic and microcosmic now lies before the eyes of the scientist in great part revealed and discovered. And why do we say *discovered*? Precisely because this system of laws is not projected into nature or constructed there by us in consequence of some innate subjective mental form; nor invented as a useful aid for economy of thought and study, to make our knowledge of things more easy; nor due simply to agreements or conventions among men of science. Natural laws exist, so to speak, incarnate and secretly operative in the heart of nature, and by observation and experiment we seek them out and discover them.

And let it not be said that matter is not a reality, but an abstraction fashioned by physical science; that nature is in itself unknowable; that our sense-world is another world apart in which the phenomenon, or mere appearance of the external world, makes us imagine the reality of the things which it conceals. No, nature is a reality, and a knowable reality. Though things appear to be speechless, and are in fact so, yet they have a language in which they speak to us, a language which springs from their secret recesses as water from a perennial source. Their language is their causality, which reaches our senses with the sight of color and movement, with the sound of metals, of whirlwinds and living beings, with the sweet and bitter of honey and gall, with the scent of flowers, with the hardness, the weight and the heat of their matter, imprinting upon us an image or likeness through which our intellect leads us to the reality of things. So, when you speak, you do not speak of the image and likeness of things which is in your mind, you speak of things themselves; and you are perfectly capable of distinguishing between the phenomenon of your sense-world and the substance of things: between the appearance of gold and gold itself, between the appearance of bread and bread itself, whose substance you eat as food, assimilating it and making it of one substance with your own body. The movement of things towards us causes a likeness in us; and without such likeness there can be no conformity of our intellect with real things, no knowledge; for nothing can be called true which has not a certain conformity with our intellect. The things from which our mind takes its knowledge are the measure of our mind and of the laws which we find in them and derive from them; but they in turn are measured by the eternal intellect of God, in which are all things created, as every work of art is in the mind of the craftsman. (6). With mind and hand the scientist discovers and reveals them, distinguishes and classifies them, and not as one who chases birds, but as one who holds them in his grasp and examines their nature and intrinsic properties. When Lothar Meyer and Mendelejew in 1869 drew up that simple scheme of chemical

(6). St. Thomas "De Veritate." Q.I; a. 2.

elements now known as the natural system of the elements, they were profoundly convinced that they had discovered an orderly arrangement founded upon their properties and internal tendencies, a classification suggested by nature, the future development of which promised the most far-reaching discoveries about the nature and constitution of matter. At the time of the discovery the thought of a so-called mental economy could not have arisen, because the original scheme showed several gaps; nor could it have been a matter of convention, since the arrangement was imposed by the qualities of matter itself. This is only one of many examples through which leading scientists, past and present, have come to be convinced that they are the heralds of a truth which is one and the same for all nations and peoples of the globe; a truth grounded essentially upon an "*adequatio rei et intellectus*," a truth which is simply the more or less perfect, the more or less complete conformity achieved between our intellect and the objective reality of natural things. In this conformity consists the truth of our knowledge.

REFUTATION OF PHENOMENALISM.

But do not be misled, like the philosophers and scientists who have thought that our cognitive faculties know only their own modifications and sensations, concluding that our intellect has knowledge only of the images received from things, and that it is therefore the images of things and not things themselves that are the object of our science and of the laws which we formulate about nature. This is a manifest error. Are not the things which you perceive and the things of which your science speaks, reasons, and argues, one and the same? Are we now speaking to you or to the images which are being formed in Our eye through seeing you present before Us? If what you perceive and know were only the images of your sensations, it would follow that all natural sciences—from the stars to the atom, from the sun to the electric light, from the minerals to the cedars of Lebanon, from microbes to man and the medicines that cure his diseases—would not be dealing with things outside the mind but only with the intelligible images which you contemplate in your mind even when you are dreaming. The sciences which exalts a Copernicus, and a Galileo, a Kepler and a Newton, a Volta and a Marconi, and other famous and deserving investigators of the external, physical world that surrounds us—this science would be nothing but a beautiful day-dream, a fanciful story of the physicists; the appearance would take the place of reality and truth of things; and to assert or deny one and the same thing would be equally true. No, science does not deal with dreams or with the images which we draw from them. As the Angelic Doctor taught, following Aristotle, the stone itself cannot be in our mind; but the stone produces an image or likeness of itself, first in our senses and then in our intellect, and thus through this likeness the stone is in our mind and in our study, so leading us back to itself and to the truth. (7)

(7). Cf. St. Thomas. I,q.76; a.2, ad.4.

Recent investigations in the field of experimental psychology prove or rather confirm, our conviction that these images are not merely the product of an autonomous subjective activity, but psychic reactions to stimuli independent of the subject and arising from things themselves; reaction which conform to the different qualities and properties of things, and vary with the varying of the stimulus.

Hence, the images which, whether through light, heat, sound, taste, smell, or in any other way, are imprinted by things upon our sense organs and through the internal senses reach the intellect, are nothing else than the instrument provided by nature, our first teacher, to make herself known to us. But it is none the less true that we are able to examine, study, investigate this instrument; we are able to reflect upon these images and what they tell us of nature, and upon the manner in which they impart to us knowledge of the world that surrounds us. From the act by which our intellect knows the stone, we pass to the act of knowing how our intellect knows the stone; and this is the second act, not the first, because man, being born without innate ideas and without dreams of any previous like, enters into this world devoid of impressions, and without any knowledge of the world, being so made—as we have already observed—that “through the senses it doth apprehend what then is meet for the intellect to view.”

USE FOR LITHIUM

Oxygen and its destructive companions, water vapor and carbon dioxide, cause metals and alloys to decarburize and oxidize when heat treated at high temperatures. Under the stimulus of war conditions, the problem has brought out the development of Lithium, an energetic metal of the lightest non-gaseous element known to science. Its vapors fill the furnace, blanket the work, so that it will not decarburize or oxidize.

The Milvay Notebook
Chicago Apparatus Co.
2 2 1943-44

PENICILLIN BABY

“A second bactericidal substance is reported to have been isolated from the same source, to which the name ‘penatin’ has been given. While penicillin is a nitrogen-containing substance of low molecular weight, soluble in organic solvents, the new penatin is a complex protein, not dialyzable or extractable by organic solvents. The action of penatin is attributed to its ability to decompose dextrose with the liberation of hydrogen peroxide, which is toxic to bacteria. Penatin is highly bacteriostatic and bactericidal in the presence of dextrose, and is particularly effective against Gram-negative organisms not readily acted upon by penicillin. Fairly large doses of penatin have been injected successfully into experimental animals, and small amounts are even tolerated intravenously.”

J. Chem. Ed. 20 12 608.

ASTRONOMY

SOME PROBLEMS IN STATISTICAL ASTRONOMY

By REV. FRANCIS J. HEYDEN, S. J.

Eratosthenes held that the earth was a sphere and from a simple observation he made a remarkably good determination of its circumference. His theory was not accepted as a fact, however, until eighteen centuries later Magellan's ship actually circumnavigated the earth. Nor was that achievement by any means the one scientific experiment from which we have learned subsequently all that we know about the size, shape and general structure of this planet which we inhabit.

By comparison it may be said that our galaxy or Milky Way system has already had its Magellan. From a very careful study of the space distribution of 93 globular star clusters Shapley discovered that they formed a frame-work around the outside of the stars of the Milky Way. The globular cluster system defines the outer limits of the galaxy, and thus gives us a picture of its overall size and shape. Hence it seems right to say that Shapley has figuratively circumnavigated the Milky Way.

At present our galaxy appears to be a massive dynamical system of stars, lenticular in shape, about 30,000 parsecs* in diameter, perhaps 3000 parsecs in thickness, and with our sun situated about 8000 parsecs away from its center. These dimensions compare quite favorably with those of our neighboring galaxy in Andromeda, about 1,000,000 light years away. When we put ourselves in a position in that galaxy corresponding to our distance from the center of our own system, we are amazed to find that we are out in the faintest and dimmest portion. We are far from the hub of our own universe.

This eccentric position of the sun is impressed rather strongly on the careful observer who merely studies the surface appearance of the Milky Way on a clear moonless night. In the regions around Auriga, Taurus and Gemini the surface brightness is clearly less impressive than in the regions near the direction of the galactic center is Sagittarius. The general illumination of the Milky Way, however, does not arise from stars visible to the naked eye, but from the stars fainter than eleventh apparent magnitude. Hence one cannot fully appreciate the intricate details of its apparent structure without the aid of long exposure photographs. On such a photograph one sees dark clouds intermingled with the bright star fields. These clouds suggest at once the presence of huge masses of non-luminous material which

* 1 parsec = 3.26 light years.

are absorbing and blocking off the light of the stars behind them. These obscuring clouds lend variety and beauty to the general appearance of the Milky Way, but they are shutting off the astronomer's view of his universe.

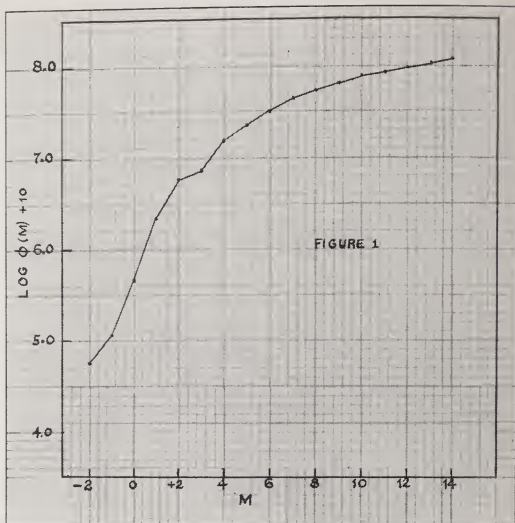
The globular clusters have provided a frame-work within which the galaxy is suspended. They provide no information concerning its structural details. When we look at some of the other galaxies in space around us it is impossible to say from our present knowledge which one is exactly typical of our own. Does our galaxy throw out its stars in sweeping spiral arms, or is it one of the more compact types? We cannot fly off into space and look back at our stellar system. Our only recourse is to adopt the slow tedious procedure of boring from within and studying the structure part by part as far as our telescopes can penetrate along the entire Milky Way.

We might attempt to proceed by determining the distance and position of each star. Eventually we should be able to reconstruct the whole galaxy on a reduced scale. But it is soon discovered that the trigonometric methods of measuring the distances of stars are inadequate for the problem. Well within 50 parsecs the probable error of a single trigonometric distance is almost as great as the distance itself. Besides, the task of fixing the position in space for every star would be quite impossible, or at least impractical. There are perhaps thirty thousand million stars in our galaxy. We must have recourse to statistical methods.

In approaching a study of the galaxy from a statistical point of view the astronomer prescinds from the individual stars and looks for distribution laws which will describe the whole star system in a more or less idealized way. Since he is looking for details of real structure, he proceeds cautiously in order to avoid methods which lack sufficient flexibility and which will not represent his observed data.

One of the most important distribution laws which has required almost forty years of diligent observation and study is known as the luminosity function. It can be simply defined as the distribution function which gives the frequencies of stars of various absolute luminosities in a unit volume of space. The standard unit of volume is the cubic parsec and the function itself is set up empirically from a space survey of the stars in the vicinity of the sun. If the stars are actually distributed throughout the entire Milky Way system according to such a law, then the statistical astronomer really has a powerful tool with which he can analyze the structure of the galaxy.

Let us suppose for the present that such a function is a real law for the whole star system, and that interstellar space is perfectly transparent. Since each element of volume contains the same proportion of stars of different intrinsic brightness, the details in the surface appearance of the Milky Way will be determined only by variations in the star density at different distances in the line of sight. Apparent



magnitudes would be related to the absolute magnitudes, or intrinsic brightness, only through the inverse square law of distance. By determining the densities for the various distances it would be possible to compute the numbers of stars of successive apparent magnitudes which should be found in any square degree of the sky. In a generalized form the whole problem could be expressed in the equation:

$$A(m) = \int_0^{\infty} \omega r^2 D(r) \phi(M) dr$$

where $A(m)$ = the number of stars of apparent magnitude "m" per square degree of the sky.

$$\omega = \frac{4\pi}{41,253} = \frac{4\pi}{\text{no. sq. deg. on a sphere}}$$

r = distance in parsecs.

$D(r)$ = the density distribution in which $D(r)$ equals unity near the sun.

$\phi(M) = (m + 5 - 5 \log r)$ = the luminosity function for the vicinity of the sun, i. e., the number of stars with absolute magnitudes between M and $M - dM$ per cubic parsec in the vicinity of the sun.

The above equation states the theoretical problem. Its solution, however, will depend upon the quantities which have to be determined from observations. Two of these unknown quantities are the density function, $D(r)$ and the luminosity function $\phi(M)$. The third unknown is the amount of deviation from the inverse-square law of distance through which the apparent magnitude, "m" is related to the absolute magnitude, "M". These three quantities provide the principal problems of the statistical astronomer. We shall consider each one briefly in the following paragraphs.

We have already defined the luminosity function. Naturally this represents a distribution curve which must be derived empirically from an accurate space survey of the solar neighborhood. At first one might think that such a task is relatively simple, but when one begins to take a census of all the stars in a representative sample volume of space the range in absolute magnitudes provides a very great difficulty. Stars range in absolute brightness from about $M = -5$ to $M = +15$, a difference of 10^8 times in intrinsic luminosity. It is no wonder then that just the form of the luminosity function has proved to be a major problem. Figure 1 below gives the best value of the function which is the result of nearly forty years of research.

At least we now have a working model of the general luminosity function, even if it does not fit any analytical formula. We even know quite definitely how it varies systematically for different distances above and below the galactic plane. But we are still far from certain whether or not it varies systematically with distance or with direction in the galactic plane itself. These two problems will be settled only by future surveys of sample volumes of space at great distances from the sun. Several investigations of this nature have already shown that deviations from the local function exist out to a distance of about 300 parsecs, but these do not have a serious influence on the results of a statistical analysis of the space density of stars. The problem of establishing the universality of the luminosity function will remain unsolved until we can test its validity in sample volumes throughout the entire galaxy.

The second quantity, the density function, $D(r)$, is the real unknown in the general formula which we are trying to find. In the elaborate analytical methods used about twenty-five years ago, the density function was assumed to have an exponential or a gaussian form. The general luminosity function was also given a gaussian form, so that it was possible to compute an $A(m)$ value for each apparent magnitude. These computed values could be checked with those derived from actual star-counts. It was found that this analytical method was not flexible enough. In the case of isolated star clouds or star vacancies, which are certainly important details of galactic structure, the assumed density law would obviously fail to give a true representation of the real densities. Finally with better

determinations of the luminosity function it became clear that the gaussian distribution had to be discarded.

Before discussing the third quantity which constitutes another problem for the statistical astronomer, the numerical method by which the theoretical equation can be solved will be described.

Let us divide space around the sun into a series of concentric shells, in such a way that the radius of each successive shell will be given by the relation.

$$\log r = \frac{2k}{10}$$

where $k = 1, 2, 3, 4$, etc.

The absolute magnitudes, "M", of all the stars in any shell will be related to the apparent magnitudes, "m", through the formula,

$$M = m + 5 - k$$

The next step will be the construction of a table for the observed luminosity function in which we can see at a glance the number of stars of apparent magnitude "m" which would be found in each shell, if the space density were everywhere the same. This is a very simple procedure, as is shown in the following diagram:

Table 1.

	m 1	m 2	m 3	m 4	m 5 . . .
k 1	$a(m_1, k_1)$	$a(m_2, k_1)$	$a(m_3, k_1)$	$a(m_4, k_1)$
k 2	$a(m_1, k_2)$	$a(m_2, k_2)$	$a(m_3, k_2)$	$a(m_4, k_2)$
k 3	$a(m_1, k_3)$	$a(m_2, k_3)$	$a(m_3, k_3)$	$a(m_4, k_3)$
k 4	$a(m_1, k_4)$	$a(m_2, k_4)$	$a(m_3, k_4)$	$a(m_4, k_4)$
k 5	$a(m_1, k_5)$
		M_1	M_2	M_3	M_4 M_5

Each entry in Table 1 is the product of ϕ ($m + 5 - k$), taken from the observed luminosity function and the volume of the shell in cubic parsecs. It is clear that "k" will increase by 0.2 in the logarithm, while the logarithm of the volume increases by 0.6. With "k" increasing by numerical succession, all the values along any diagonal in the table will belong to the same absolute magnitude. Therefore, in constructing the table it is simply necessary to start at the lowest value of the apparent magnitude and work down a diagonal by adding 0.6 to the preceding value.

To use the table for the study of stellar distribution of stars in a particular region of the sky, the volume considered in each entry is reduced to the part of a shell which is enclosed by one square degree.

Then the sum of any vertical column of entries will be the total number of stars per square degree of apparent magnitude "m". This sum is the same quantity, $A(m)$, which we can observe by counting the stars on a photograph of the sky.

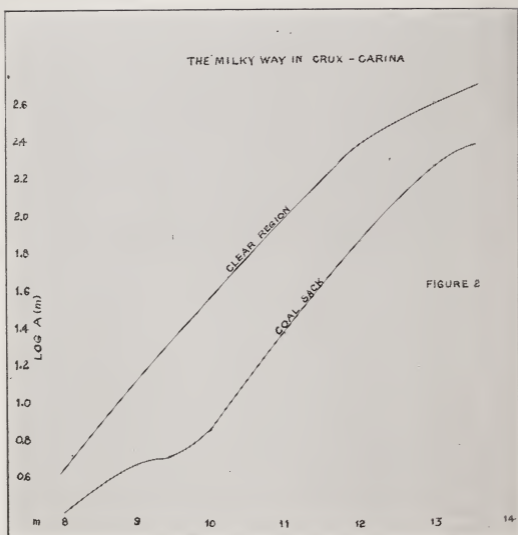
Since the table has been constructed for a constant density at all distances, the next step will be to find the factors by which the entries for each shell must be increased or decreased in order to obtain a set of computed $A(m)$ values which will match the observed $A(m)$'s. This can be done very easily by use of "trial and error". The procedure requires less time than one may anticipate, and the results are quite satisfactory. The set of factors which gives the best fit between the computed and observed data represent the relative densities, referred to the vicinity of the sun which has been taken as unity. At present there is not much need to reduce them to absolute quantities. It is sufficient to know the relative star density at different distances from the sun to arrive at a good idea of galactic structure.

The luminosity function which we have given in Figure 1 is a combination of the luminosity data derived by van Rhijn at the Kapteyn Laboratory in Holland and by Luyten at Minnesota. The numerical method of computing the space densities which has been described was designed by the Dutch astronomer, Kapteyn, nearly thirty years ago and has been used very effectively by Bok and his collaborators at Harvard.

Let us now take up the third major problem in statistical astronomy. It seems that the numerical method has given us the right of way for a study of the space density of stars. However, our fundamental relation, $M = m + 5 - k$, is based upon the inverse square law. What if there is absorbing material in interstellar space which vitiates that law? Then the computed space densities are false. We have found regions of low star density which do not actually exist. We must therefore, be sure of the transparency of space.

If one studies a long exposure photograph of the Milky Way, the dark patches and channels which appear in the midst of rich star fields suggest the existence of huge clouds of absorbing material. One sure way of determining the existence of the absorption of star light would be to determine the distance of a star in two ways; one, by trigonometry which would be independent of the absorption, and the other, by photometry, which would be influenced by the absorption. That is certainly the best method but trigonometric parallaxes are not accurate enough at the great distances at which the dark clouds lie.

For the Milky Way itself two methods are now rather common. In one an apparently clear region of the sky is selected as a standard. By plotting the logarithm of the counted number against the apparent magnitudes of the stars in the clear region a fairly straight line is



obtained. Comparing this curve with curves derived from nearby obscured regions, which are assumed to have the same number of stars, the amount of the absorption is immediately evident. Figure 2 shows two $\log A(m)/m$ curves for adjacent regions in the Southern Milky Way. The obscured region, known as the Coal Sack, may be seen on the photograph near the two bright stars of the lower portion of the Southern Cross. The counts for the clear region are taken from the part just to the right of the Coal Sack. The curve for the Coal Sack indicates quite definitely that there is space absorption caused by the dark nebula which is a naked eye object in that part of the sky.

While $\log A(m)/m$ curves are useful for detecting relative absorption between two adjacent parts of the Milky Way, the results are not certain with regard to the distance or the absolute amount of the absorption. The so-called "clear" region may be also affected by interstellar material and this can be ascertained only by some method which is independent of the star-counts.

At present the best practical method for determining the amount and the distance of interstellar obscuration takes advantage of the selective nature of the absorption. There is clear observational evidence that interstellar material consists of gases which manifest themselves as absorption lines in the spectra of stars. Besides the absorption lines due to the gases there is another type of selective absorption which manifests itself in the effective color of a star, and which has been found to vary approximately with the reciprocal of the wavelength. This effect requires the presence of very small dust particles about the size of the wavelength of light. No doubt there are also large particles or chunks of material in interstellar space which simply block off the light in all wavelengths by an equal amount, but observations indicate that practically all of the interstellar absorption arises from the fine dust which affects the colors of the stars. This is a great advantage, for by comparing the observed colors of stars in the Milky Way with the normal colors which they should have a quantity known as the color-excess can be obtained. This color excess is a measure of the total absorption between the observer and the star.

By determining the total absorption in this way the solution of the statistical equation for star densities is again possible. Table 1 which is used for the numerical solution is corrected for the observed absorption by shifting the entries for the different space shells to the right by an amount corresponding to the observed absorption. The density analysis by trial and error can then be carried out.

The absorption problem is not disposed of as easily as the above description may suggest. The main difficulty lies in the tremendous distances involved. For star-counts down to the 14th apparent magnitude the space densities must be computed out to 10,000 parsecs. Reliable measures of the color excesses of stars reach out to only about

2000 parsecs. Since the spectral type of these stars must be known, one of the future problems of the larger and new type telescopes will be the observation of the spectra of faint stars.

In the meantime another method of attack is being planned. The variable stars of the δ Cepheid type can be identified at distances of several hundred thousand parsecs by the periods of their light variation. If we can derive the color excess of these variable stars we should be able to penetrate to the greatest possible depths of the galaxy. We still lack the observational data for these stars. Since their normal color also varies with their light fluctuations it will be a difficult task to obtain all of the information necessary for a color survey with Cepheid variables. We are, however, ready to make the attempt and have ardent hopes that our results will carry us to a further knowledge of the distribution of stars in our own Milky Way.

HOW THESE CHEMISTS LOVE ONE ANOTHER!¹

"I passed a current of Chlorine through a solution of Manganese acetate, $\text{MnO} \cdot \text{C}_2\text{H}_3\text{O}_2$ under direct influence of sunlight. After 24 hours I found in the liquid a superb crystallization of a violet-yellow salt. This upon analysis proved to be, $\text{MnO} \cdot \text{C}_2\text{Cl}_6\text{O}_2$. When heated to 110° in a current of chlorine, it was converted with liberation of oxygen gas into a new golden yellow compound, supposed to be $\text{MnCl}_2 \cdot \text{C}_2\text{Cl}_6\text{O}_2$. This new substance dissolved with the aid of heat in pure chloral, and this liquid, not attacked by chlorine, was used to continue the treatment with chlorine. I passed dry chlorine into it for four days, keeping the liquid always near the boiling point. During this time a white substance was constantly deposited which upon careful examination was recognized as MnCl_2 . The liquid was cooled sometime afterwards when no more precipitation was observed, and a third substance was obtained in small greenish yellow silky needles. It contained no Manganese and was shown to be, $\text{Cl}_2 \cdot \text{Cl}_2 \cdot \text{C}_2\text{Cl}_6\text{O}_2$. On acting again with Cl_2 , on an aqueous solution of this substance, carbonic acid (CO_2) was set free, and on cooling it to 2° it deposited a yellow mass formed of small plates closely resembling chlorine hydrate. Carbon had, in fact, been replaced by Chlorine, which was now the only element present in the compound. The Mn acetate had been converted by substitution into pure chlorine, which however had a vapor density which indicated the presence in the molecule of not fewer than 25 atoms of chlorine. $\text{Cl}_2 \cdot \text{Cl}_2 \cdot \text{Cl}_6 \cdot \text{Cl}_2 \cdot \text{H}_2\text{O}$."

S. C. H. WINDLER.

¹This is a letter published by Leibig, probably in his *Analen*, as a communication from the London Chemical Society. The purpose was to ridicule Dumas' substitution theory. It was probably actually written by Woehler. A footnote was appended as follows;

"I have just learned that there are already in the shops of London fabrics of spun Chlorine, much sought after in hospitals, and preferred to all others for night-caps, underwear, etc;."

BIOLOGY

A FEW VITAMIN COMPLEXES

By JAMES J. DEELEY, S. J.

In these days of radio broadcasts and high pressure advertisements there are few people who have not heard of and are not aware of the need of vitamins in their daily diet. Some advertisers have even invaded the realm of the comic strip in order to impress all manner of people with their infallible cure of all ills. There can be no doubt that the people in general have become increasingly vitamin conscious within the last two decades. An elderly gentleman of my acquaintance, who has been and still is subject to many ills, recently informed me and I quote, "I would be a much healthier man today, only I have no vitamins." This homely expression merely serves to indicate how vitamins have assumed the proportion of a health complex through the medium of the radio and advertisement. However, few of these men of high business acumen, to say nothing of their popular audience, realize how recent have been the discoveries and developments in this subject, their so-called panacea of all ills. For the sake of increased business and profit they neglect to tell their audiences that the spark plugs, no matter how high their caliber, are useless if the machine is otherwise out of order.

The fact that human beings are subject to various diseases as a result of deficient diet has long been known. Scurvy occurred among the crusaders of the thirteenth century. In 1520 the Austrian surgeon Kramer pointed out that scurvy could be cured by the addition of oranges, limes and lemons to the diet and the British Navy first introduced lime juice as a preventive of this disease in 1795. The fatalities and disabilities caused by the old time nervous disease beri-beri was greatly decreased when the Japanese medical officer Takaki investigated and improved the diet of the sailors.

The 1911 Casimir Funk, while investigating the cause of beri-beri, obtained a crystalline substance from rice polishings which was capable of preventing or curing this nervous disorder. He called the substance *vitamine* because of its evident importance to life. He believed that it was an amine. Subsequently a number of similar substances playing a similar role in nutrition were discovered, to which the general term *vitamin* was given, the individual vitamins being designated by the letters of the alphabet. This nomenclature had the decided advantage of being non-committal with regard to their chemical nature and the properties of these substances. This, perhaps, is one of the reasons why so many of the older physicians viewed with sus-

picion the value of the much heralded vitamin as late as the nineteen twenties.

The following is a classification of the known vitamins and their chief physiological effects . . .

Vitamin A (antixerophthalmic)	(B ₁ or thiamin (antineuritic)
	(
Vitamin B Complex	(riboflavine
	(B ₂ complex (nicotinic acid
	(vitamins B ₃ to B ₆
Vitamin C (antiscorbutic)	
Vitamin D (antirachitic)	
Vitamin E (antisterility)	
Vitamin K (antihemorrhagic)	

Vitamins A, D, E and K are fat-soluble. The others, namely, those of the B complex and C are water-soluble. Riboflavine is also called vitamin B₂ and sometimes vitamin G.

VITAMIN A

This vitamin was discovered as a result of the investigations of Hopkins in England and Osborne and Mendel and McCollum and Davis in America. Its chief sources are mammalian and fish liver, egg yolk, butter, cream and a number of vegetable foods. Cod-liver oil has a very high but halibut oil a much higher content in this vitamin. Vegetable oils, beef fat, cereals with the exception of maize are relatively poor in vitamin A content. The sweet potato rather than the white, carrots, spinach, string beans, green peas, bananas and cantaloupe are rich sources of vitamin A, whereas cabbage and radishes have it present in small amounts or have none at all.

Chemically vitamin A is soluble in fats and fat solvents. It is present in the unsaponifiable fraction of the fat. It is resistant to heat in the absence of air but is readily destroyed by oxidation at all temperatures. Though colorless it gives a blue coloration with antimony chloride in the presence of oxygen. It is an unsaturated alcohol with the empirical formula $C_{20}H_{30}O$ and is derived from betacarotene. Vitamin A has been isolated in the crystalline form and has recently been synthesized. Euler in 1928 showed that pure carotene was capable of replacing vitamin A in the diet. When Moore later experimented with rats and fed them pure carotene, he found that the vitamin content of the liver was greatly increased. As a result of these and other experiments it is concluded that carotene and possibly other plant pigments are the precursors of vitamin A. When taken in food, the carotene or provitamin, as it is called, is converted into vitamin A in the liver (probably in the Kupffer cells).

This vitamin apparently does not exist as such in plants but only in the form of provitamin. Algae, diatoms and other marine plants

synthesize the provitamin; they in turn serve as food for small marine forms such as molluscs and copepods; the molluscs etc. serve as a source of supply of provitamin for young cod and herring and other small fish; these in turn supply larger fish, such as halibut, with vitamin A. Herbivorous animals obtain their supply of provitamin from alfalfa which is a particularly rich source. Man can therefore obtain a supply of this vitamin either by consuming plant foods rich in provitamin or animal tissues which contain vitamin A preformed.

A lack of vitamin A in the diet is first evidenced by the individual's failure to gain weight. However, this is no indication that growth in the true sense has ceased. It has been proven that skeletal growth continues even though the individual exhibits no gain in weight. In man one of the earliest manifestations of this dietary deficiency is a dryness of the skin followed by papular eruptions due to changes in the hair follicles and atrophy of the sweat and sebaceous glands, with consequent suppression of their secretions. In the human subject often the epithelial linings of the respiratory, alimentary and urinary tracts and the ducts of various glands tend to become converted into stratified squamous epithelium with a consequent drying up of their secretions. Night blindness or failure to regenerate visual purple after the eyes have been exposed to strong light is another indication of vitamin A deficiency. With regard to this there is an interesting example of custom anticipating science. In tropical climates where this disease flourishes the natives treat the eyes with liver poultices and also add liver to the diet. Some observers have declared that vitamin A deficiency results in degenerative changes in the nervous system. Mellanby and Greene have spoken of vitamin A as the antiinfective vitamin although other investigators have observed no increased immunity to infection as a result of additions of vitamin A to the diet. Only in cases where there was previously a vitamin A deficiency in the diet has there been any noticeable increase in antiinfection activity when vitamin A was added to the diet.

VITAMIN B COMPLEX

The first vitamin to be discovered was called the antineuritic vitamin, since its absence from the diet was proven to be the cause of beri-beri. When the nomenclature of referring to the vitamins by letter was adopted the antineuritic vitamin was called vitamin B. Later investigation proved it to be a complex of vitamins, which today is known as the much heralded vitamin B complex.

VITAMIN B₁ OR THIAMIN

Beri-beri is a disease which for centuries has been prevalent in Japan, China, India, the Dutch Indies, and the Philippine Islands. It is characterized by inflammation of the peripheral nerves which leads to a progressive paralysis of the limbs and sensory disturbances. The first definite indication that the cause of this disease is dietetic in na-

ture can probably be found in the log-books of the Japanese navy. In 1885 Takaki, a Japanese medical officer of the fleet, revised the standard diet and practically eradicated the disease from the service. In 1890 Eijkman, working in his laboratory in the Dutch West Indies, observed a fowl which seemed to exhibit many of the symptoms that humans suffering from beri-beri exhibit. He also found that the fowl had been existing on a diet of polished rice. Other investigators, using his data, soon proved that the condition in fowl and beri-beri in man are closely allied and can be cured by the addition to the diet of rice polishings or an extract prepared from them. In later years as a result of this polished rice gave way to the unpolished type in the rice-eating countries of the East.

The vitamin B₁ requirement is influenced by several factors, e. g., the nature of the diet, the quantity of food consumed, and the basal metabolism. It is increased by a diet high in carbohydrate concentration and reduced by one high in fat content. It is now recognized that B₁ exerts a specific influence on growth. Loss of appetite and consequent undernutrition is, however, a contributory factor in the retardation of growth resulting from deficiency of this vitamin. In infants failure to gain at a normal rate, though the diet appears adequate, is considered in some instances to be due to a low B₁ intake. Loss of appetite is an early symptom of this deficiency, being evident some time before polyneuritis appears. It is probable that Anorexia is, in part at least, secondary to the relaxed state of the gastro-intestinal musculature, the suppression of the gastric juices (gastric and pancreatic) and the reduction of the motor activity of the gastro-intestinal tract all of which are important features of this deficiency. It is also probable that many diets that are thought to be adequate are below the optimal level and that gastro-intestinal abnormalities are not uncommonly the result of mild grades of deficiency. Loss of appetite in children and adults in many instances is due to the same cause. Obviously a revision of the diet by the addition of foods rich in vitamin content is indicated.

VITAMIN B COMPLEX (ORIGINALLY CALLED THE HEAT STABLE, ANTI- DERMATITIS OR PELLAGRA-PREVENTING (P. P.) FACTOR)

The great majority of substances which are rich in B₁ content contain other factors of the B Complex. The solubilities of B₁ and B₂ are similar. For this reason it was not for some time after the water-soluble B vitamin had been discovered that any suspicion arose that it was a complex and contained vitamins other than the anti-neuritic factor. In 1919 Mitchell observed that certain substances rich in B₁ content had little or no power to promote growth. This suggested the existence of two vitamins. The final proof of this was furnished by later investigators. These latter investigators found that when brewer's yeast had been autoclaved, thus removing the antineu-

ritic factor completely, and was added to the diet of animals, the animal continued to grow for a while but later died. When an alcoholic extract of maize was added to the diet as the sole source of the vitamin, there was again a short period during which the animals gained in weight, but this ceased and they soon died. However, when both were combined the result was a normal growth. It is clear then that the factor necessary for growth is present in autoclaved yeast and another is present in the alcoholic extract of maize. Two vitamins necessary to growth must, therefore, exist in close association to one another. The growth factor in autoclaved yeast is B₂, that in maize is B₁.

B₂ itself is a complex containing several factors. Three of these factors which are of special interest are, namely, riboflavine or lactoflavine, nicotinic acid and vitamin B₆. Riboflavin is responsible for part of the growth promoting property of the B₂ complex but it has not antidermatitis action. Nicotinic acid in all probability, acts like riboflavine, acting as a constituent of a coenzyme serving as a hydrogen transporter in the respiratory system of cells. It exhibits no action on the autonomic nervous system and is relatively non-toxic. Little is known as to the role played by B₆ in human nutrition, though certain symptoms in pellagrins and those suffering from beriberi which are not relieved by nicotinic acid, thiamin or riboflavine are abolished in a rather dramatic fashion by pure B₆. The lack of this complex in humans results in arrested growth and the maize eaters' disease, pellagra. Vitamins other than those mentioned are known to be contained in the B complex but much less is known concerning their functions and properties. They are referred to as B₃, B₄, and B₅. These factors have some significance in the growth and the weight conditions of animals. What their significance may or may not be in human nutrition is not definitely known.

These are merely a few of the facts and probabilities of only two of the vitamins, A and B. The data is meager but valuable. It is to be hoped and no doubt it is already true that our scientists laboring in the East and in all corners of the globe, wherever our armies are, are and will bring forth new and important developments that have been dictated by the need of a vast army of men and women at war.

HOW DO YOU EXPLAIN IT?

Why is the product of two negative quantities a positive quantity?

The minus sign has two meanings; it may be the symbol of operation (subtraction), or it may be a symbol of quality. Thus, in $9-7$, the "minus" is a symbol of operation, i.e., it indicates that the number 7 is to be subtracted from the number 9 leaving a remainder of 2.

But in the expressions, $-a$ or -7 , the minus is a symbol of quality. Moreover, though 7 is a number, -7 is not a number but a quantity, or we may say, -7 is a combination of a number and a quality, which is not a number. Similarly the plus sign may be a symbol of operation (addition) or of quality. Thus, plus and minus mean in one connection "above and below", in another "to the right and to the left" and in a third "possession and privation".

In general, therefore, we may say that the minus sign means a REVERSAL of relation. The "unit of reversal" is indicated by (-1) . Applying this as a MULTIPLIER reverses the relation indicated by the quantity "plus a ". If we apply it (-1) a second time we get back to the original quantity, i.e., two reversals of the same relation applied to the same thing brings the relation of these things back to what it was originally. Thus, if we have between two things the relation of above and below, and reverse this once we get the relation of below and above, and reversing this again we get back to the relation above and below from which we started.

FIRESIDE ENTERTAINMENT?

Works designed to teach Chemistry by experiment are already in use, both here and abroad, but most of them take for granted the possession of expensive apparatus, and a laboratory; scarcely any are designed to bring the practical study of the science within the means of the more elementary schools;—and none are to be found suited to the winter-evening firesides all over the country, where the younger and more advanced of both sexes would delight in chemical experiments, did not the apparently necessary expense of apparatus forbid them.

E. N. Horsford in the Introduction to
The Principles of Chemistry, illustrated
by simple Experiments, by
Dr. Julius Adolph Stoeckhardt.
Translated by C. H. Pierce, M. D. 1854.

Nth Derivative of Finite Products

by

C. F. Koehler, S.J.

The purpose of this article is to serve as a ready reference for those who may have occasion to use n^{th} derivations of finite products (or Leibniz's Rule, as it is called in its generalized form).

Elementary Calculus books derive the first derivative with respect to x of the product of two functions (uv) and state the rule:

$$(d/dx)uv = v(du/dx) + u(dv/dx)$$

The form of this result we can simplify by the use of operational notation

$$(d/dx)uv = D_{uv}uv = (D_u + D_v)uv$$

where $D_u = d/dx$, i.e., D_u is an operator which operates on u , treating all other variables as constant. The same explanation holds true for D_v .

The operators D_u , D_v , D_w ,..... and their characteristics can be found in Cohen's "Differential Equations"⁽¹⁾ and are summarily restated here.

- 1) Symbolic Operators behave like algebraic quantities for the processes of addition, subtraction, and multiplication.
- 2) The symbolic operators are commutative in nature.
- 3) They can be applied to any finite number of operations.

We now extend the derivative of a product of two functions to include the n^{th} derivative of the product. This extension depends upon the theorem of Mathematical Induction ⁽²⁾ and can be briefly shown as follows:

.

- (1) - "Differential Equations" - revised edition - by Abraham Cohen - pg. 109, no. VI, 6.
- (2) - "A Treatise on Advanced Calculus" - Philip Franklin - pg. 2, no. 1.

$$(d/dx)uv = (D_u + D_v)uv$$

$$\begin{aligned}(d^2/dx^2)uv &= (d^2u/dx^2)v + 2du/dx dv/dx + (d^2v/dx^2)u \\ &= (D_u^2 + 2D_u D_v + D_v^2)uv \\ &= (D_u + D_v)^2 uv\end{aligned}$$

Consequently, the rule for higher derivatives takes the form of the corresponding power of the operators.

Assume

$$(d^m/dx^m)uv = (D_u + D_v)^m uv \quad (2)$$

and prove that (d^{m+1}/dx^{m+1}) takes the form of $(D_u + D_v)^{m+1} uv$.

To do this, take the derivatives of both sides of (2).

$$d/dx(d^m/dx^m)uv = D_{uv}(D_u + D_v)^m uv$$

$$(d^{m+1}/dx^{m+1})uv = (D_u + D_v)(D_u + D_v)^m uv$$

$$\text{where } D_{uv} = (D_u + D_v)$$

$$\text{or, } (d^{m+1}/dx^{m+1})uv = (D_u + D_v)^{m+1} uv \quad (3)$$

Therefore, if in equation (2) we replace m by $m+1$, we obtain the same result as (3), and since the rule is true for $m = 1, 2$, then it will be true for $n = m$ or the generalized form now is:

$$(d^n/dx^n)uv = (D_u + D_v)^n uv$$

$$\text{where } n = 1, 2, 3, \dots$$

$(D_u + D_v)^n$ can be expanded by the binomial theorem because of the properties of D_u and D_v stated above.

However, in the Binomial Expansion $D^0 u = u$.⁽¹⁾

In the elementary Calculus it is shown that the first derivative of a product of n functions can be derived by the following rule:⁽²⁾

$$(d/dx)uvw\dots z = vw\dots z (du/dx) + \dots\dots\dots$$

which can again be expressed by operational symbols as

$$(d/dx)uvw\dots z = (D_u + D_v + D_w + \dots + D_z)uvw\dots z$$

The actual proof for the 1st, 2nd, 3rd derivative of $uvw\dots z$ can be shown to be equivalent to the form

$$(D_u + D_v + D_w + \dots + D_z)^1$$

$$(D_u + D_v + D_w + \dots + D_z)^2$$

$$(D_u + D_v + D_w + \dots + D_z)^3$$

The method is simple but tedious and the reader can easily verify the truth of the statement. The proof of the generalized form, that is, that

$$(d^n/dx^n)uvw\dots z = (D_u + D_v + D_w + \dots + D_z)^n uvw\dots z$$

follows the identical line of argument (by the use of Mathematical Induction) as employed above for the n^{th} derivative of the product of two functions.

Now $(D_u + D_v + D_w + \dots + D_z)^n$ can be expanded by the Multinomial Theorem⁽³⁾

The rule for the Multinomial Theorem is given by

$$\sum \frac{n!}{a!b!c!\dots m!} u^a v^b w^c \dots z^m$$

where $a + b + c + \dots + m = n$.

.

(1) - cf. Advanced Calculus - Wilson - pg. 11, eq. 44.

(2) - cf. Granville - Smith - Longley, "Elements of the Calculus" - Revised Edition - no. 35.

(3) - Algebra - G. Chrystal - Vol. II, Second Edition, pg. 15.

As an example expand $(D_u + D_v + D_w)^3$. (here $n=3$)

The Multinomial Theorem states:

$$(D_u + D_v + D_w)^3 = \sum \frac{3!}{a!b!c!} D_u^a D_v^b D_w^c$$

and by convention $0! = 1$.⁽¹⁾ Therefore substituting all possible permutations of 0, 1, 2, 3 whose individual sums equal 3 we obtain:

$$(D_u + D_v + D_w)^3 = 1) \frac{3!}{(0!1!2!)=2!} D_u^0 D_v^1 D_w^2 = 3u D_v^1 D_w^2$$

$$2) \frac{3!}{0!2!1!} D_u^0 D_v^2 D_w^1 = 3u D_v^2 D_w^1$$

$$3) \frac{3!}{1!0!2!} D_u^1 D_v^0 D_w^2 = 3D_u^1 D_w^2$$

$$4) \frac{3!}{1!2!0!} D_u^1 D_v^2 D_w^0 = 3D_u^1 D_v^2 w$$

$$5) \frac{3!}{2!1!0!} D_u^2 D_v^1 D_w^0 = 3D_u^2 D_v^1 w$$

$$6) \frac{3!}{2!0!1!} D_u^2 D_v^0 D_w^1 = 3D_u^2 v D_w^1$$

$$7) \frac{3!}{3!0!0!} D_u^3 D_v^0 D_w^0 = D_u^3 v w$$

$$8) \frac{3!}{0!3!0!} D_u^0 D_v^3 D_w^0 = u D_v^3 w$$

$$9) \frac{3!}{0!0!3!} D_u^0 D_v^0 D_w^3 = u v D_w^3$$

$$10) \frac{3!}{1!1!1!} D_u^1 D_v^1 D_w^1 = 6 D_u^1 D_v^1 D_w^1$$

Summing:

$$(D_u + D_v + D_w)^3 = D_u^3 v w + 3D_u^2 v D_w^1 + 3D_u^2 D_v^1 w + 3D_u^2 v D_w^2 + 3D_u^2 v D_w^2 + 3u D_v^2 D_w^2 + 3u D_v^2 D_w^2 + u D_v^3 w + u v D_w^3 + 6D_u^1 D_v^1 D_w^1 \quad \underline{\text{q.e.d.}}$$

.....

(1) - Chrystal's Algebra, Vol. II, pg. 4.

The author wishes to acknowledge the help and encouragement given him by Stanley J. Bezuska, S.J.